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Treatment of Anchorage of Main Bars in Reinforced Concrete by Codes of Practice -A Critical Review
(Part One: Straight Bar Anchorages)

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ABSTRACT---- ***This paper compares important provisions and parameters related to anchorage of straight bars in reinforced concrete of BS8110, EC2 and ACI-318 as well as empirical equations by by Orangun et al , Darwin et al. and Batayneh are also considered in a parametric study.***

The study investigates the parameters which have influences on the bond and anchorage as there are some of them have not been considered by all of them and found their significance in obtaining more accurate bond resistance like transverse pressure, transverse reinforcement, the yield strength of the transverse reinforcement, bar geometry, anchorage length, concrete cover and transverse pressure.

A parametric study and test results from literature are used in demonstrating the treatments by them and significant commentaries are given.

Keywords--- straight anchorage, bond stress,anchorage length ,concrete strength, concrete cover, transverse reinforcement, transverse pressure

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1.INTRODUCTION

There is a considerable debate about the treatments of bond and anchorage in codes of practice. The debate rooted in their approach in understanding the factors that influencing firstly the mechanism of transferring force between the ribbed bars and the surrounding concrete and secondly the bond failure in the surrounding concrete to the anchored bars.

These factors which include : the strength of the concrete, the ratios of covers and bar spacings to the bar diameter,the local properties of the concrete adjacent to the bar, which are affected by the position and orientation of the bar relative to the direction of concreting, the ratio of the bond length to the bar diameter in end anchorage or pull- out situations, the details of the transverse reinforcement crossing potential failure surfaces transverse pressure from reactions the details of the bar ribs the size of the bar, given that scale effects often arise where concrete is subjected to non - uniform tension.

The bond of contemporary ribbed bars relies on the bearing of the ribs on the surrounding concrete. This bearing produces outward radial forces and , for normal ratios of cover to bar size , bond failure involves splitting of the concrete cover. It has often been found that at failure small wedges of concrete remain locked in position ahead of the ribs. As the thickness of cover increases the failure surface around the bar changes and becomes a continuous cylinder with a diameter equal to that of the ribs. Splitting failure remains possible as the actions on this failure surface are shear and radial compression with the latter requiring tension in the cover. Eventually, for very large covers, bars may be extracted, without splitting the cover.

It is clear from the above that bond resistance should be expected to be influenced by the thickness of the concrete cover to a bar. It is also reasonable to expect influences from transverse reinforcement crossing the surface at which failure occurs and from transverse pressure acting at a support.

In most structural members the maximum tension in the main bars is reduced at a rate controlled by the shear on the member and the shear reinforcement provided, leaving only a part of the tension to be absorbed by the end anchorages of the bars.Within the end anchorages the rate of the reduction of bar forces is not externally controlled but depends upon the relationship between bond stress and slip (movement of the bar relative to the surrounding concrete). Slip is greatest at the end where the bar forces are greatest . At least initially the bond stresses are therefore greatest at the same end and decrease toward the free ends of the bars. Splitting can be initiated at the loaded ends and may well produce a progressive failure, throughout which the average bond stress is always below the maximum bond strength per unit length.

There are no publications where the codes based on them in derivation of their expressions. It is necessary to carry out a parametric evaluations and using tests from literature to:

1. Investigate the differences between these codes with respect to the concerned parameters mentioned above and,
2. Judge equations by Orangun, Darwin and Batayneh in terms of their accuracy and Comparisons with the codes.

2.ANCHORAGE REQUIREMENTS

The load effect at any section in the main bars required by a member's design for bending and shear and where relevant axial load and torsion. The resistance is the force at the same section, that can be developed by bond and any mechanical anchorages such as end hooks or welded cross bars.
For slender beams with deigned stirrups, the structural model by EC2[2] is a truss, such as that shown in Fig.(1) for a region in which the shear force and shear reinforcement are constant. The force T in the tension chord can be found by considering rotational equilibrium at section 1-1 is $T = \frac{M}{z} + \frac{1}{2} V \cot\theta$

while for vertical equilibrium at section 2-2 is $V = A_{sv} f_{sv}\, z \cot\theta / s$

where A_{sv} is the area of one model of stirrups representing the total area of stirrups in a length s and f_{sv} is the stress in the shear reinforcement at the design ultimate limit state. θ is the angle of the web compression and can, subject to the limit on the concrete compression by chosen freely in the range $1 \le \cot\theta \le 2.5$.

The first equation gives a simple solution for the distribution of the chord force, which is correct for most of any shear span.

An alternative way of obtaining the force T can be derived by considering a vertical section 3-3 at a distance $(z \cot\theta) / 2$ in the direction of increasing moment from section 1-1. The moment at section 3-3 is $M + \Delta M$ where $\Delta M = V (z \cot\theta) / 2$ and $(M + \Delta M)/ z = M / z + V (\cot\theta) / 2$ which is the same as the force in the first equation. Thus the design tensile force for the bottom chord can be obtained by the 'shift rule', i.e. by shifting the M / z diagram through a distance equal to $(z \cot\theta) / 2$ in the direction of decreasing moment.

The chord force obtained from a case-specific truss model, the first equation and the shift rule are generally similar, provided the angle of web compression at the level of the tension chord is constant.

When a concentrated load act at the high moment end of a simply supported shear span and is applied to the top surface, the web compression radiates from beneath the load. $Cot\theta$ reduces from its value in the remainder of the shear span to area at the loaded section and the tension chord force at this section is M / z .In these circumstances the value of T from the question has to be limited to $T \le M_{\max} / z$ .If there is any doubt about the variation of the force in the section chord, it should be calculated from a truss model of the whole shear span.

The excess of T over M / z , for a given loading on a given beam with vertical stirrups, is proportional to $\cot\theta$ and can thus be reduced by decreasing $\cot\theta$ in the de $\cot\theta$ sign range $1 \le \cot\theta \le 2.5$, although the benefit in terms of anchorage conditions is obtained at the cost of increasing the amount of shear reinforcement require. Another way of decreasing the excess of T is to use inclined rather than vertical stirrups.

The forces considered above are those in tension chords. If part of the main reinforcement is outside the web breadth in a tension flange, the force in the reinforcement is increased by an amount equal to the longitudinal components of the forces in the compression members of the horizontal truss system by which the changes of longitudinal forces arising from the web truss are transmitted to the bars in the flange outstands- see Fig.(1)-

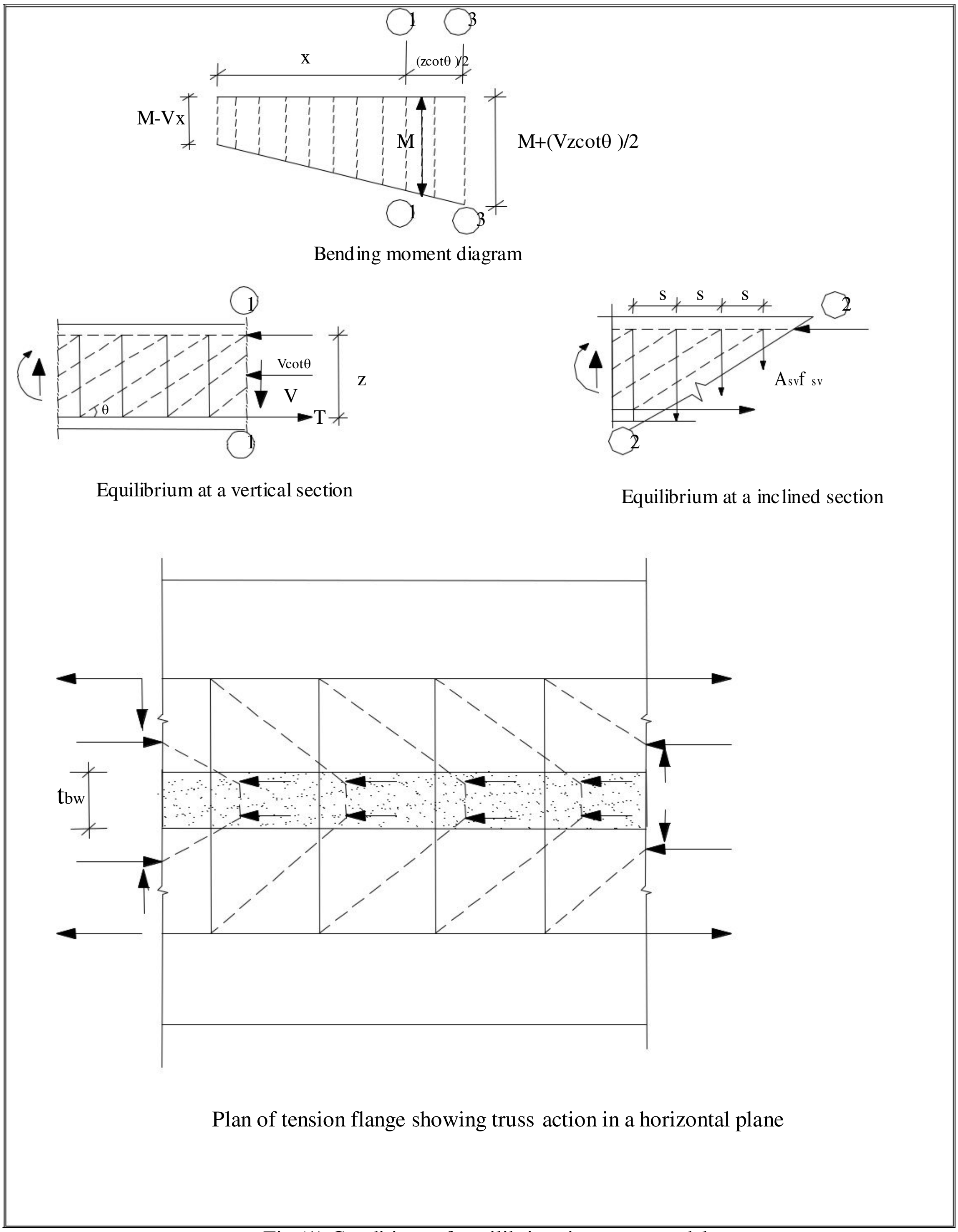


Fig.(1) Conditions of equilibrium in a truss model

In slender members without designed shear reinforcement, the tension chord force can be increased above M / z inspite of the absence of web truss action. Factors involved in this probably include inclinations of flexural cracks, the development of some arching due to the bending of the concrete teeth between flexural cracks and compression across flexural cracks induced by vertical displacement across crack faces. EC2 allows for an increase of the chord force by applying a shift equal to d to the M / z diagram, although it seems likely that the shift should be a function of the shear.

It might be argued that the tension force in the bars should be very low in areas where there are no flexural cracks, but it is not usual to rely upon the longitudinal tension in the concrete because of the possibility of there being significant tension due to restrained shrinkage.

In short shear spans loaded fm above and supported from below, even in the absence of shear reinforcement, the shear resistance can be above the shear cracking load. this is due to the possibility of the loading being carried o a support directly by inclined compression, i.e. a tied arch action which depends on the end anchorage of the reinforcement. Truss models, suitable for designs relying on this action, are shown in Fig.(2) .The two models for short shear spans with stirrups give same results in terms of the tensions at supports.

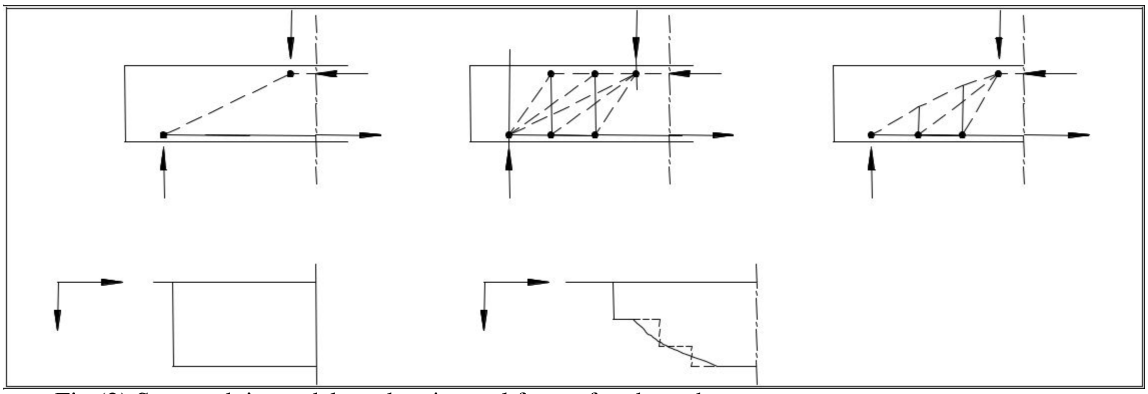


Fig.(2) Strut and tie models and main steel forces for short shear spans

The paragraphs above show how the resistance required of the tension chord can be obtained from a design for shear and bending. It remains to show how the resistance available from bond and, where relevant, mechanical anchorage can be assessed.

For any individual bar, anchored by bond alone, the design yield force can be developed in a length l_d , such that

$$\int_0^{l_d} \pi\varphi f_{bd}\, dl = F_{yd}$$

if there is a mechanical anchorage, such as a hook, at the end of a bar

$$\int_0^{l_d} \pi\varphi f_{bd}\, dl = F_{yd} - F_{hd}$$

where F_{hd} is the design force developed by the hook, f_{bd} is the design bond stress for a length dl , and l_d is measured from the loaded end of the hook.

The bond stress f_{bd} is a function of the concrete strength, the bar's cover and spacing from other bars, the transverse pressure and the transverse reinforcement although not all of these factors are taken into account in all design recommendations. The ways in which the various factors are treated vary in different approaches. In some, all of those considered are expressed within f_{bd} , but in others including EC2, some of them are taken into account by factors applied to l_d . If, as in EC2, the bond stress is taken to be constant along any part of the l_d where there are no changes of cover, transverse pressure at the potential development of F_{yd} is as illustrated in Fig.(3).

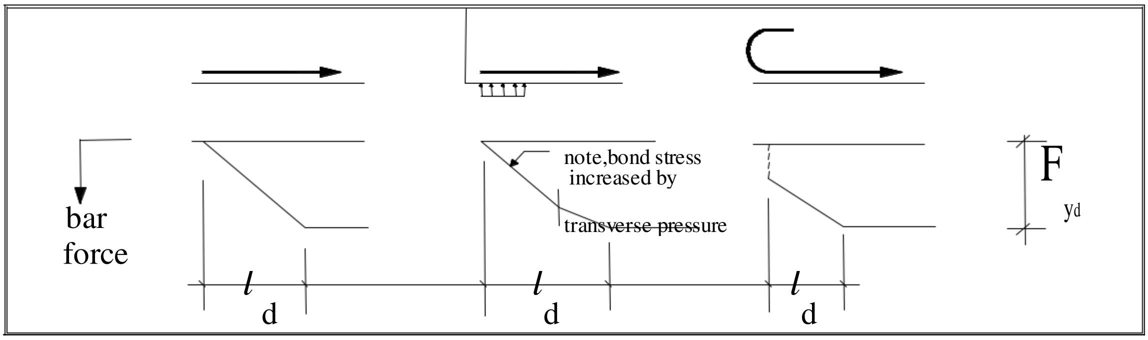


Fig.(3) Example of bar force development in anchorages

The total resistance available from the main steel of a member can be obtained as the sum of the potential resistances from the individual bars. This can be compared with the resistance required and needs to exceed it at all points.
Fig.(4) shows this for an example of a simply supported beam subjected to a series of concentrated loads approximately a uniform distribution of loading. The form of the diagram is similar to one inn EC2. The bars ‘ a ’ continue to the end of the beam and their anchorage length is reduced by the effect of the transverse pressure from the support. The bars ‘ b ’ and ‘ c ’ are curtailed in the shear span, where there is no transverse pressure, and their development lengths are longer.
The force that can be developed by the main steel is equal to the force required by the truss model (T) at four places: i)at the inner edge of the support, i) and iii) at the locations of the curtailments of bars ‘ b ’ and ‘ c ’ and iv) in the region of maximum moment.

The main purpose of a diagram of this type is to determine where bars can be curtailed. The situation at the inner face of the support defines the number of bars that have to be continued to the end of the beam. Thereafter the intersections of the line for (T) with horizontal line at F_{yda} and $F_{ydb} + F_{yda}$ determine the locations at which bars can be curtailed.

The figure presents a slightly special case in that the development lengths of the three groups of bars do not overlaps. If they do, the diagram becomes a little more complicated, but there is not difficulty in principle in its construction.

In practice the line for the required chord force will often be an envelope of the lines from several load cases, but this presents no special problems.

It is possible that if transverse reinforcement is stressed due to the shear on a beam, the stresses developed in it may be higher than those taken into account in the estimation of l_d which assumes the transverse bars to be stressed only the effects of bond. The higher stresses could be expected to improve the bond of bars in the corners of stirrups. There are however difficulties in making allowance for such an effect. One is that the stresses in stirrups resisting shear have their maxima at the levels, where the stirrups are intersected by inclined cracks, and may be significantly reduced by bond in the lengths between shear cracks and the level of the main bars. Another problem arises if the design of anchorages is based on an envelope of chord forces, as it would be difficult to identify the stirrup stresses relevant to the individual load cases.

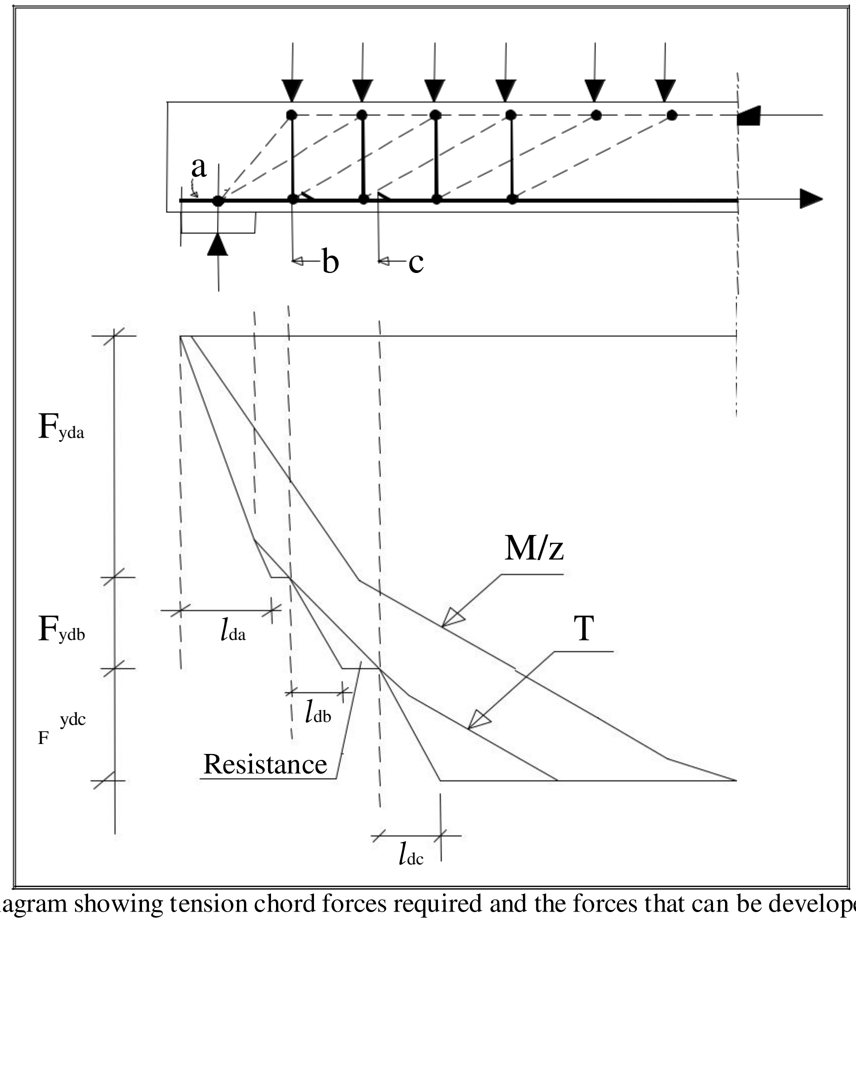


Fig.(4) Diagram showing tension chord forces required and the forces that can be developed by bond

3.CODE OF PRACTICE RECOMMENDATIONS

3.1BS 8110 [1]

In the British code BS8110:2005, the design ultimate bond stress f_{bd} for bars with a minimum cover of at least one bar diameter(φ) and a minimum clear spacing also at least one bar diameter, is equal to $k\sqrt{f_{cu}}$, where k is a constant depending on the type of bar and whether the bar is in tension or compression and f_{cu} is the cube strength of the concrete.
The design tensile force that can be developed in a bar is :

$$F_{sd} = k\sqrt{f_{cu}}\,.\pi.\varphi.l_{b,eff} \quad \text{……………………..(1)}$$

For straight bars the effective anchorage length $l_{b,eff}$ is the distance from the bar end to the section at which F_{sd} is considered .

The design value of k for normal type 2 deformed bars in tension is 0.5 which corresponds to a characteristic value of 0.7. The code applies k =0.5 to bars in beams only if minimum links are provided . In the absence of minimum links the design value of k is 0.35. For bars in slabs k =0.5 whether or not there are links.This could well be interpreted as meaning that k=0.5 is all right for interior bars with or without minimum links, but requires links around corner bars.

For anchorages with end hooks or bends , BS8110 generally requires checks on both bond and bearing stresses.So far as bond is concerned , the design limit for the bar force at the start of the bend is:

$$F_{Rd\,1} \leq \pi.\varphi.l_{b,eff}\,.f_{bd} \text{……………………..(2)}$$

where : f_{bd} is the same as for straight anchorages and $l_{b,eff}$ is in general the length of the bend plus that of the tail. However the following lengths may be used if greater

3.2Euro Code 2:2004 [2]

EC2 considers most of the parameters which have influences on bond resistance in reinforced concrete structures such as concrete strength, position and orientation during casting, anchorage type, concrete cover, bar spacing, transverse reinforcement and transverse pressure.

The Code defines a basic design ultimate bond stress for ribbed bars as:

$$f_{bd} = 2.25\eta_1\eta_2 f_{ctd} \quad \text{……………………….. (3)}$$

Where: f_{ctd} design tensile strength of concrete, η_1 is a coefficient related to the quality of the bond condition and the position of the bar during concreting, η_2 = is a coefficient which considers bar diameter, $\eta_1 = 1.0$ where good conditions are obtained. e.g. for bottom bars and for top bars where there is no more than $250mm$ of fresh concrete below the bars or where the bars are more than 300mm from the top, $\eta_1 = 0.7$ for other cases e.g. bars more than 250mm from bottom (and less than 300mm from the top if $h > 600mm$) and for all bars in structural elements built with slip forms, η_2 =1.0 for $\varphi \leq 32mm$ and $\eta_2 = (132-\varphi)/100$ for $\varphi > 32mm$

The basic anchorage length is defined as :

$$l_{b,rqd} = \frac{\varphi}{4}\frac{f_{sd}}{f_{bd}} \quad \text{………………………. (4)}$$

Where: f_{sd} design stress of a bar for the ULS at the position from where the anchorage is measured. The design anchorage length l_{bd} can be calculated from:

$$l_{bd} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\, l_{b,rqd} \geq l_{b,\min} \quad \text{………………… (5)}$$

where :

l_{bd} = design anchorage length, α_1 = the effect of the form of the bars (assuming adequate cover), α_2 = the effect of concrete cover. $c_d = \min(c_b, c_s, s/2)$, α_3 = the effect of confinement by transverse reinforcement, α_4 = the influence of one or more welded transverse bars along the design anchorage length l_{bd}, α_5 = the effect of pressure transverse to the plane of splitting along the design anchorage length, α_1 =1.0 for straight bars,

$\alpha_2 = 1 - 0.15(c_d - \varphi)/\varphi$ for straight bars, $\alpha_2 = 1 - 0.15(c_d - 3\varphi)/\varphi$ for other than straight bars - $\alpha_2 \geq 0.7$ and ≤ 1.0,

α_3 =1- $K\lambda - \alpha_3 \geq 0.7$ and ≤ 1.0 where : $\lambda = \left(\sum A_{st} - \sum A_{st,\min}\right)/A_s$, A_s = area of a single anchored bar with the maximum bar diameter , $\sum A_{st}$ = cross-sectional area of the transverse reinforcement along the design anchorage, length. (note A_{st} =area of a transverse bar, $A_{st,\min}$ cross-sectional area of the minimum transverse reinforcement . =0.25 A_s for beams and 0 for slabs, $K = 0.1$ for a bar in the bends of stirrups, $K = 0.05$ for a bar with transverse bars in its cover, $K = 0$ for a bar in the cover to transverse bars, α_4 is not relevant in the present context and is taken as 1.0, $\alpha_5 = 1 - 0.04p$.

In addition to the individual limits on $\alpha_2 \alpha_3$ and α_5 the product of α_2 , α_3 and α_5 is limited to ≥ 0.7 and ≤ 1.0 , p = transverse pressure (N/mm^2), $l_{b,\min}$ = the minimum anchorage length if no other limitation is applied: for anchorage in tension : $l_{b,\min}$ > maximum of (0.3 $l_{b,rqd}$;10φ ; 100mm),for anchorage in compression $l_{b,\min}$ > maximum of (0.6 $l_{b,rqd}$;10φ ; 100mm)

From the above , ignoring $l_{b,\min}$

$$l_{bd} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 \left(\frac{\varphi\,\sigma_{sd}}{9\eta_1\eta_2 f_{ctd}}\right) \qquad \text{.............................(6)}$$

Where $f_{ctd} = f_{ctk,0.05}/15 = 0.14 f_{ck}^{2/3}$ for $f_{ck} \leq 50N/mm^2$

In effect equation (6) corresponds to design and characteristic bond strengths

$$f_{bd} = \frac{2.25\eta_1\eta_2}{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} f_{ctd} = \frac{0.315\eta_1\eta_2}{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} f_{ck}^{2/3} \qquad \text{.............................(7)}$$

and $$f_{bk} = \frac{2.25\eta_1\eta_2}{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} f_{ctk} = \frac{0.4725\eta_1\eta_2}{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} f_{ck}^{2/3} \qquad \text{..............................(8)}$$

For f_{ck} $50\,N/mm^2$, $f_{ctk} = 1.48\ln(1 + f_{cm}/10)$ where f_{cm} is the mean cylinder strength and can be taken as $f_{ck} + 8\,N/mm^2$, and $f_{ctd} = 0.99\ln(1 + f_{cm}/10)$. However bond stresses greater than those for $f_{ck} = 60\,N/mm^2$ should not be used, " unless it can be verified that the average bond strength increases above this limit".

3.3ACI 318-2005[(3)]

ACI 318-05 treats the anchorage requirements for straight deformed bars in tension by giving the required development length as :

$$l_{bd} = \frac{f_s}{1.1\sqrt{f_c}} \cdot \frac{\Psi_t.\Psi_e.\Psi_s\,\lambda}{(c_d + \varphi/2 + K_{tr})}\varphi^2 \quad \geq 300mm \qquad \text{.........................(9)}$$

in which $c_d + \varphi/2 + K_{tr}$ may not be taken as more than 2.5 φ , f_s is the bar stress to be developed, f_c is the cylinder compression strength of the concrete $\leq 70MPa$, ψ_t =1.3 for horizontal reinforcement placed such that more than 300 mm of fresh concrete is cast below the development length or splice. ψ_t =1.0 for other situations, ψ_e treats epoxy-coated reinforcement. Its value for uncoated bars is 1.0 and this used in the following, ψ_s =0.8 for $\varphi \leq 19mm$ or 1.0

for $\varphi \geq 22mm$ (ψ_s =0.8 can probably be assumed for $\varphi = 20mm$), λ =1.3 for lightweight aggregate concrete and 1.0 for normal density concrete, c_d = the lesser of the smaller cover and $s/2$

$K_{tr} = A_{tr} f_{yt} / 10 s_t n$, A_{tr} = total area of transverse reinforcement ,within the spacing " s_t ", that crosses the plane of splitting through the reinforcement being developed, s_t = spacing of transverse reinforcement, n = number of bars being spliced or developed along the plane of splitting.

The definition of K_{tr} is not very clear. It does not define ' the plane of splitting' or how to treat A_{st} if a stirrup crosses a crack plane twice. It probably intends the plane to be assumed to be horizontal so that all the vertical legs of stirrups count.

Equation (9) corresponds to a limiting bond stress for bars without epoxy coating of :

$$f_{bd} = \frac{0.275\sqrt{f_c}\left(c_d + 0.5\varphi + K_{tr}\right)}{\psi_t \psi_s \lambda \varphi} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

There is no explicit strength reduction factor ϕ (approximately a partial safety factor for resistance) in this section of ACI 318-05.The commentary states that 'An allowance for strength reduction is already included in the expressions for determining development and splice lengths. To judge from the values of ϕ for other circumstances, that for bond would be about 0.8 corresponding to $\gamma_m = 1.25$.

4.DISSCUSSION AND PARAMETRIC STUDY

The three codes' equations for design bond stresses for the ultimate limit state are given below as applied to straight uncoated tension bars, with deformations complying with the requirements of EC2 or ACI 318, used in concrete of normal density and without welded cross bars. Where a code gives an expression for the bond length required to develop a given steel stress, the design bond strength has been obtained as $f_{bd} = \varphi f_{sd} / 4 l_b$.

1- BS 8110 $f_{bd} = 0.5\sqrt{f_{cu}}$ ………………(11)

for slabs and for beams with minimum links. Otherwise $f_{bd} = 0.28\sqrt{f_{cu}}$

2- EC2 $f_{bd} = \dfrac{2.25\eta_1\eta_2}{\alpha_2\alpha_3\alpha_5} f_{ctd}$ ………………(12)

with ($0.7 \leq \alpha_2\alpha_3\alpha_5 \leq 1.0$) and $f_{ctd} = 0.14 f_{ck}^{2/3}$ for $f_{ck} \leq 50\, N/mm^2$

3-ACI 318 $f_{bd} = \dfrac{0.275\sqrt{f_c}}{\psi_t\psi_s}\left(\dfrac{c_d}{\varphi} + \dfrac{1}{2} + \dfrac{K_{tr}}{\varphi}\right)$ ………………(13)

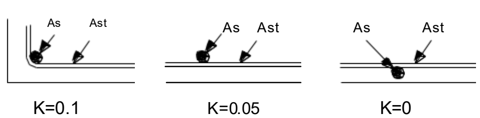
with $\left(\dfrac{c_d}{\varphi} + \dfrac{1}{2} + \dfrac{K_{tr}}{\varphi}\right) \leq 2.5$

The treatments of the influences of influential parameters by the above equations are summarised in table (1).

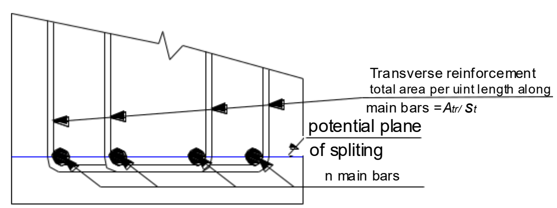
Table (1) Parameters included in bond strength design by BS8110 , ACI-318 and EC2

Code	Concrete strength	Top bar effect	φ (mm)	Cover (mm)	A_{st} (mm^2)	p $\left(\frac{N}{mm^2}\right)$
BS8110	$\sqrt{f_{cu}}$ $f_{cu} \le 40\ (N/mm^2)$			(1)		
EC2	$f_c^{2/3}$ $f_c \le 50\ (N/mm^2)$ for $f_c > 50$ See section 2.2.2	$\eta_1 = 1.0$ btm (2) $\eta_1 = 0.7$ top	$\eta_2 = 1.0$ for $\varphi \le 32mm$ $\eta_2 = (132-\phi)/100$ for $\varphi > 32mm$	$\alpha_2 = 1 - \frac{0.15(c_d - \varphi)}{\varphi}$ c_d is the min.of $(c_b, c_s, s/2)$	α_3 =1- $K\lambda$ where : $\lambda = \frac{(\sum A_{st} - \sum A_{st,\min})}{A_s}$ $\sum A_{st,\min}$ =0.25 A_s	$\alpha_5 = 1 - 0.04p$ ≥ 0.7 ≤ 1.0
ACI-318	$\sqrt{f_c}$ $f_c \le 70\ (N/mm^2)$	$\psi_t = 1.0$ btm(3) ψ_t =1.3 top	ψ_s =0.8 for $\varphi \le 19mm$ ψ_s =1.0 for $\varphi \ge 22mm$	$c_d + 0.5\varphi$ c_d is the min.of $(c_b, c_s, s/2)$	$K_{tr} = \frac{A_{tr} f_{yt}}{10 s_t n}$	

(1) The min. c_b/φ, c_s/φ and $(s/2)/\varphi$ are limited to=1.0 (2) In EC2 a bar is considered a top bar if it is more than 250mm above the bottom and less than 300mm below the top of the wet concrete during casting. (3)In ACI 318 a bar is considered a top bar if it is more than 300mm above the bottom of the wet concrete during casting.For additional limits see text.



(a) Values of K for beams and slabs by EC2



(b) Definition of Ast by ACI-318

Fig.(5)shows the relationships between f_{bd} and f_{ck} for bars with minimum (negligible) transverse reinforcement. Fig.(5-a) is for bottom-cast bars and Fig.(5-b) is for top-cast bars. In both parts there are single lines for BS8110 and two lines for EC2 ,one for $c_d = \varphi$ and one for $c_d \geq 3\varphi$, beyond which there is no increase in f_{bd}. There are four lines for ACI 318, a pair for $c_d = \varphi$ and a pair for $c_d = 2\varphi$ beyond which there is no further increase in f_{bd}. In each pair there is one line for $\varphi \leq 19mm$ and one for $\varphi \geq 22mm$. The effect of bar size is not shown for EC2 in which f_{bd} is independent of φ for $\varphi \leq 32mm$ and then decreases by 18% when $\varphi = 50mm$.

The lines for BS8110's expression are drawn for f_{ck} up to 55 N/mm^2, although the code is not intended for use with such high strengths.For EC2 and ACI-318 the lines continue to the relevant upper limits of f_{ck} =90 and 70 N/mm^2 respectively ,although EC2 limits the values of f_{ck} to be used in calculating f_{bd} to 60 N/mm^2 " unless it can be verified that the average bond stress increases above this limit".

The differences caused by the different relationships between f_{bd} and f_{ck} are small if the upper limits on f_{ck} are ignored. This can be seen from the table(2) which gives the ratios between f_{bd} calculated from $\sqrt{f_c}$ (BS8110 and AC1-318) and those calculated from f_{ctd} (EC2) if the values of f_{bd} are scaled to be equal when f_{ck} =35 N/mm^2.

Table (2) Ratios of f_{bd} from $\sqrt{f_c}$ to f_{bd} from f_{ctd}

f_{ck} (N/mm^2)	20	25	30	35	40	50	60	70	80	90
Ratios of f_{bd}	1.10	1.06	0.93	1.00	0.98	0.94	0.96	0.98	1.00	1.02

For bottom bars with $c_d = \varphi$ the three codes' values of f_{bd} are very similar , when the bar size is 19 mm or less. If the bar size is increased to 22-32 mm, the ACI values fall distinctly below those of BS8110 and EC2, and it is only when $\varphi = 50mm$ that ACI-318 and EC2 again give similar bond stresses. BS8110 values are higher than EC2 ones for

$\varphi \; 32mm$. If the cover is increased the relationship between ACI-318 and EC2 is practically unaffected at $c_d = 2\varphi$ but EC2's f_{bd} increases by a further 21% for $c_d = 3\varphi$ while the ACI value remains as for $c_d = 2\varphi$. Since BS8110 does not make any allowance for the influence of cover,its bond stresses fall progressively lower in relation to the others as c_d / φ is increased.

The effects of increasing c_d / φ are as below:

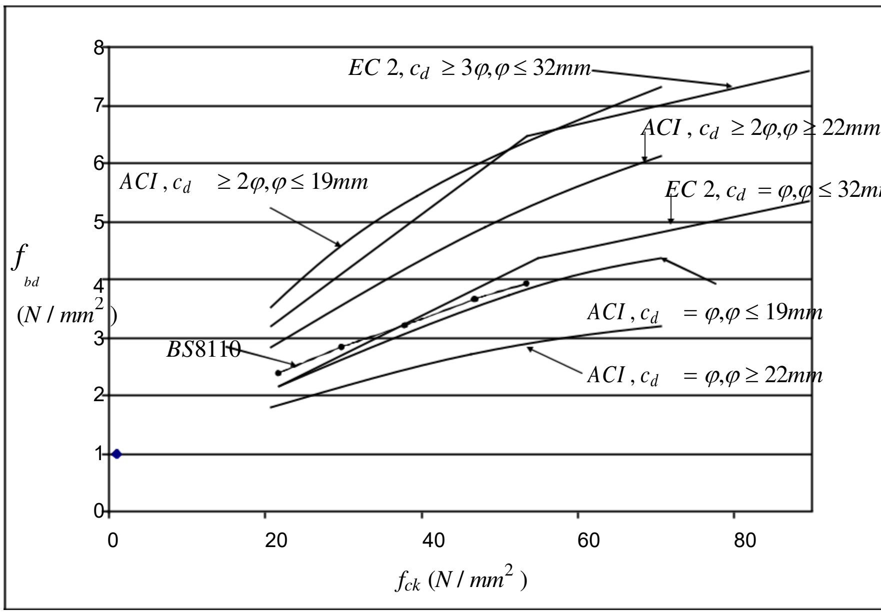
	f_{bd} for $c_d = 2\varphi$ / f_{bd} for $c_d = \varphi$	f_{bd} for $c_d = 3\varphi$ / f_{bd} for $c_d = \varphi$
EC2	1.176	1.429
ACI 318	1.667	1.667

For top-cast bars with $c_d = \varphi$, ACI-318 and EC2 give practically identical f_{bd} values if $\varphi \leq 19mm$, while for $\varphi \geq 22mm$, the ACI f_{bd} is lower, though a little less so than for bottom –cast bars, due to ACI-318's slightly higher value for the top-cast/bottom cast ratio. As BS8110 makes no allowance for a top-bar effect. Its values of f_{bd} are about 40% higher than those of both other codes when $\varphi \leq 19mm$ and about 75% above ACI-318's values for $\varphi \geq 22mm$. The BS8110 values become similar to the other codes when $c_d \approx 2.5\varphi$.

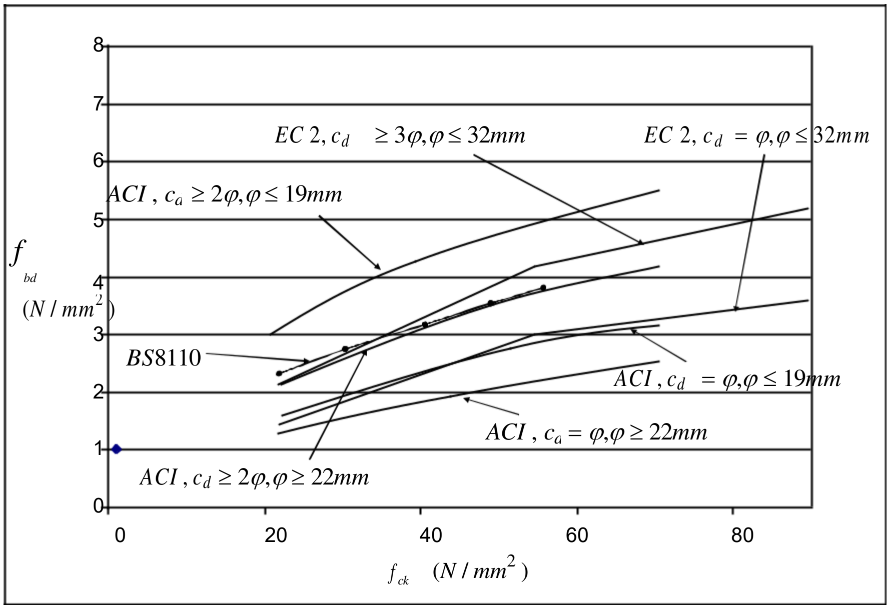
In overall terms the design resistances from EC2 and ACI-318 are similar.There are inevitably differences between the results from BS8110 and the other two codes due to its neglect of the influences of cover, bar position at casting and bar size. For bottom-cast bars the only cases where BS 8110 is significantly on the non-conservative side of the other two codes arises with minimum cover and large bar sizes.

For top-cast bars, BS8110 is significantly on the non-conservative side of ACI-318 and EC2 in all cases where $c_d \leq 2.5\varphi$, and is very much so for large bars. It is peculiar that the BS calls for increased cover for lap splices of top bars, but not for their anchorages.

A point of potential concern in all three codes is that while their expressions for bond strength are based on a minimum $c_d / \varphi = 1.0$, they allow detailing with spacings between bars as low as φ which makes $c_d / \varphi = 0.5$.



a) bottom –cast bars



b) top –cast bars

Fig.(5) Design ultimate bond stresses for bars with negligible transverse reinforcement, comparisons of BS8110,EC2 and ACI-318

Although it does require minimum stirrups to be provided in beams, BS8110 does not relate bond strength to the area of stirrups provided.

ACI-318 and EC2 do both take account of transverse reinforcement in their expressions for f_{bd}. In ACI-318 the relevant term in the equation is

$$\left(0.5 + \frac{c_d + K_{tr}}{\varphi}\right) \cdot \frac{1}{\psi_t \psi_s} \quad \text{with} \quad \left(0.5 + \frac{c_d + K_{tr}}{\varphi}\right) \le 2.5.$$

Where $K_{tr} = A_{tr} f_{yt} / 10 s_t . n$ in N and mm units, and ψ_t and ψ_s are as above.

As c_d should not be less than φ ,the maximum increase possible from transverse reinforcement is 67% when $c_d = \varphi$ but falls to 25% for $c_d = 1.5\varphi$ and to zero for $c_d = 2\varphi$.

In EC2 the equivalent term is $\dfrac{\eta_1 \eta_2}{\alpha_1 \alpha_2 \alpha_3 \alpha_5}$ with $\alpha_2 \alpha_3 \alpha_5 \ge 0.7$, $\alpha_3 = 1 - K\left(\sum A_{st} - \sum A_{st\,min}\right)/A_s$ where $K = 1.0$ for bars in the corners of links, 0.5 for main bars inside transverse horizontal bars and zero for main bars in the cover to transverse bars. η_1 and η_2 are as before.

For straight bars, $\alpha_1 = 1.0$, $\alpha_2 = 1 - 0.15(c_d / \varphi - 1) \ge 0.7$, $\alpha_5 = 1 - 0.04\,p$ for anchorages subjected to transverse pressure. If $\alpha_5 = 1.0$, then, with $c_d / \varphi = 1.0$, α_3 can be as low as 0.7 allowing a 43% increase of bond strength due to transverse reinforcement, but this reduces to 21% for $c_d / \varphi = 2.0$ and to zero for $c_d / \varphi = 3.0$.

The presence of $\psi_t \psi_s$ and $\eta_1 \eta_2$ in these expressions results in the absolute increments of f_{bd} being greater for bottom-cast, than for top-cast bars, and greater for smaller rather than large main bars. The absolute increments are also greater for higher concrete strengths.

In ACI-318 the expression includes the yield strength of the transverse reinforcement, while that in EC2 does not. The numerical influence of this difference is rather limited as the range of yield strengths covered by EC2 is only from 400 to 600 N/mm^2.

The relationships between the bond strength and the area of transverse steel per unit length are very different in the two codes. ACI-318's K_{tr}/φ is directly proportional to $A_{tr}/n\varphi s_t$, but EC2's α_3 depends on $A_{st}/\varphi s_t$ multiplied by (l_b/φ). As a result the extra bar stress that can be produced by the addition of transverse reinforcement is a linear function of $(A_{tr} f_{yt}/n\varphi s_t)(l_b/\varphi)$ in ACI-318 but a function of $(A_{st}/\varphi)(l_b/\varphi)^2$ in EC2.

To exemplify the practical implications of these recommendations, calculations have been made for bottom-cast 32 mm corner bars (making $A_{tr}/n = A_{st}$) with 32 mm cover with $f_{ck} = 35\ N/mm^2$, and zero transverse pressure. The small cover was chosen to provide the maximum scope for transverse steel to increase f_{bd}. The use of a fairly large size of main bar allows a large range of $A_{st}/\varphi s_t$ to be realistic but does have the effect of reducing f_{sd} for zero or negligible stirrups in ACI-318 .If the main bar size were reduced to $\leq 19mm$ the ACI-318 f_{sd} values for negligible stirrups would be increased by 25% and would be similar to those for EC2.

Fig.(6) shows the stresses f_{sd} that can be developed by anchorage lengths from 10 to 40 φ.With the ACI-318 approach significant increases in f_{sd} can be achieved at all values of (l_b/φ) and the stirrups needed for this are not excessive (φ_{st} =12 mm at 180 mm centres). Following EC2 the increments of f_{sd} are insignificant when (l_b/φ)=10 or 20. With (l_b/φ)=30 or 40 values of f_{sd} similar to the maxima from ACI-318 are achievable but only with very large amounts of stirrups (φ_{st} =12 mm at about 60 mm centres). In relation to anchorages at simple supports or at concentrated loads at the ends of the cantilevers, EC2 will give practically no advantage to design including transverse reinforcement , but ACI-318 may give it some advantage.

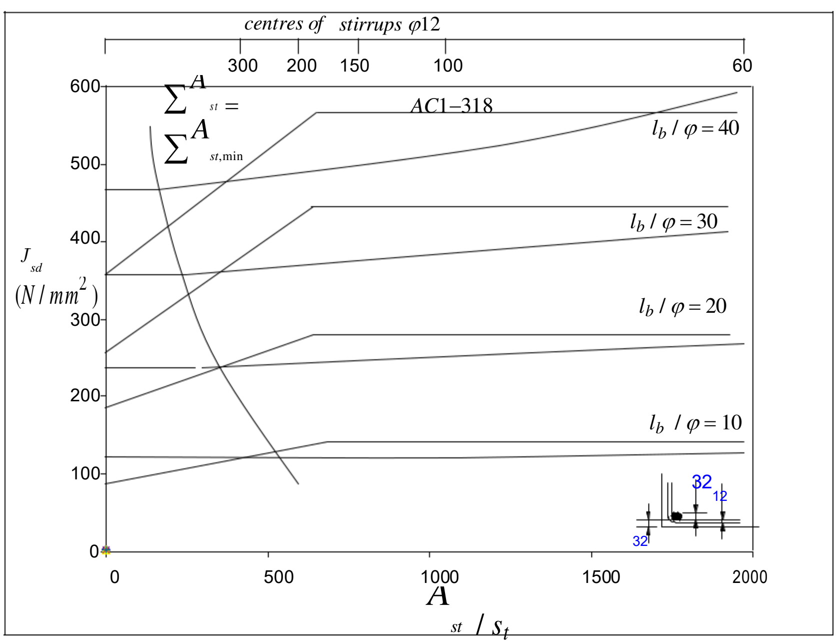


Fig.(6) Effect of stirrups on bar stresses developed by various bond lengths

(bottom-cast , corner bars $\varphi 32$, $f_{ck} = 35N/mm^2$, $f_{yt} = 500N/mm^2$, $c_s = c_b = 32mm$)

5.EMPIRICAL EQUATIONS AND EFFECTS OF PARTICULAR PARAMETERS

The various empirical equations have to be judged primarily in terms of their accuracy and comparisons with test results are presented in chapter 3.There is however some interest in the differences between their structures and the relationships between their implications and those of the more theoretical methods. The listing of equations that follows tries to express them in comparable terms, except that the codes aim to give characteristic resistances, while the equations from research are more likely to be aimed at mean strengths.

Neither lightweight concrete nor epoxy coated bars are considered.

BS8110 [1] $f_{bk} = 0.78\sqrt{f_c}$ …………..(14)

EC2 [2] $$f_{bk} = \frac{0.4725\eta_1\eta_2 f_c^{2/3}}{\left[1.15 - \frac{c_d}{\varphi}\right]\left[1 - K\left(\frac{A_{st}\, l_b}{A_s\, s_t} - 0.25\right)\right]\left[1 - 0.04p\right]}$$ …………..(15)

For $f_{ck} > 50N/mm^2$, the top line becomes $3.34\eta_1\eta_2 \ln(1.8 + 0.1 f_{ck})$, but values of $f_{ck} > 60N/mm^2$ should be used only where there is evidence that the expression is applicable. $\eta_1 = 1.0$ for bottom bars or 0.7 for top bars. $\eta_2 = (132 - \varphi)/100 \le 1.0$ .In beams $K = 0.1$ for bars in the corners of links and 0.05 for other bars. In slabs $K = 0.05$ where the transverse bars are in the cover of the main bars and $K = 0$ in the opposite case. For slabs "0.25" is replaced by 0. Each bracketed term in the bottom line and the product of the three terms are limited to lie between 0.7 and 1.0.

ACI 318 [3] $$f_{bu} = 0.34\sqrt{f_c}\left[\frac{c_d}{\varphi} + 0.5 + \frac{\sum A_{st} f_{yt}}{10 s_t n\varphi}\right] \cdot \frac{1}{\psi_t} \cdot \frac{1}{\psi_s}$$ …………..(16)

This assumes a capacity reduction factor of 0.8 has been used in determining the design expressions given in the code. $1/\psi_t = 1.0$ for bottom bars or 0.7 for top bars. $1/\psi_s = 1.25$ for $\varphi \le 19mm$ and 1.00 for $\varphi > 22mm$.

$$\left[\frac{c_d}{\varphi} + 0.5 + \frac{\sum A_{st} f_{yt}}{10 s_t n\varphi}\right] \le 2.5$$

Orangun et al. [4] $$f_{bu} = \alpha_{c,o}\left[\left(0.1 + 0.25\frac{c_d}{\varphi} + 4.15\frac{\varphi}{l_b}\right) + \frac{\sum A_{st} f_{yt}}{6020 s_t n\varphi}\right]\sqrt{f_c}$$ …….(17)

This expression is for bottom bars, for top bars f_{bu} is reduced by 25% to 30%. $\alpha_{c,o}$ is given as a step function which can be approximated by $\alpha_{c,o} = 1 + 2c_s/c_b . \varphi \le 1.64$.

$c_d / \varphi \le 2.5$, $l_b / \varphi \ge 10$, $\sum A_{st} f_{yt} / 6020 s_{st} n\varphi > 2.5$

Darwin et al. [5] $$f_{bu} = \left[\alpha_{c,D}\left(0.48\left(\frac{c_d}{\varphi} + 0.5\right) + 12.8\frac{\varphi}{l_b}\right) + 13.8 t_r t_d \frac{\sum A}{s_t n\varphi}\right] f_c^{1/4}$$ …(18)

$$\alpha_{c,D} = \left(0.1\frac{c_M}{c_m} + 0.9\right) \le 1.25 \quad , t_d = 1 + 0.1\varphi \quad , t_r = 1 + 34 f_R$$

$$\alpha_{c,D} = \left(\frac{c_d}{\varphi} + 0.5\right) + 2.75\, t_r\, t_d \frac{\sum A_{st}}{s_t\, n\varphi} \le 3.75 \quad or \quad 4.0$$

Valid only for bottom bars and $l_b / \varphi \ge 16$.

where t_r is a term representing effect of relative rib area, t_d is a term representing effect of bar size.

Batayneh[6] $\quad f_{bu} = 0.215 f_c^{2/3} \left(1 + 0.6 \frac{c_d}{\varphi}\right) \le 0.86 f_c^{2/3}$ ……………..(19)

Batayneh derived his equation for the case $c_b = c_s$ and the extension of it is c_d

where $c_d = \min(c_b, c_s, s/2)$

Equation (18) is modified by the authors and it includes the influence from transverse reinforcement and the relative rib area of the main steel and also changes the influence of concrete strength from proportionality to $f_c^{1/2}$ to proportionality to $f_c^{1/4}$ .However, as explained below, these innovations are not thought to be helpful.

Darwin et al's $f_c^{1/4}$ was introduced to improve predictions for high strength concretes, but the justification of it by Fig.1 of (reference 44) seems very weak. It shows that using the conventional $f_c^{1/2}$ slightly underestimates f_{bu} when f_c is below $25 N/mm^2$, gives good predictions for $25 \le f_c \le 50 N/mm^2$ and gives some unsafe results for $f_c > 70 \; N/mm^2$ .The figure appears to include no data for $50 < f_c < 70 N/mm^2$.

EC2's use of $f_c^{2/3}$ follows from splitting failures being tensile and EC2 taking f_{ct} to be proportional to $f_{ck}^{2/3}$ for $f_{ck} \le 50 N/mm^2$. While this reasoning is not necessarily incorrect it does ignore the quite strong possibility that ductility in tension increases with decreasing compressive strength and that this could permit a greater redistribution of stresses along the line of a crack.

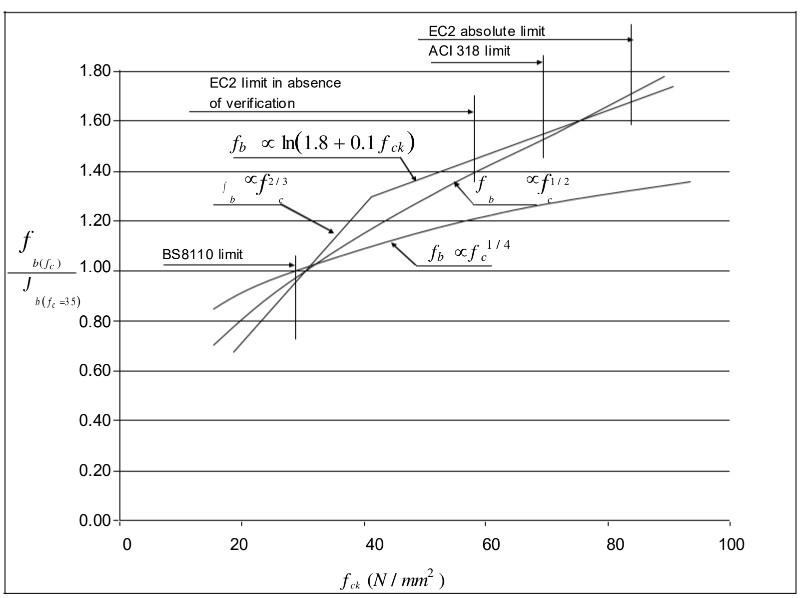


Fig.(7) Comparison of different treatments of the relationship between bond strength and concrete cylinder strength

Fig.(7) compares the effects of different treatments of the influence of f_c on f_{bu}, with all treatments scaled to give the same bond strength for $f_{ck} = 35 N/mm^2$, and shows that the use of $f_c^{1/2}$ gives results similar to EC2.Thus unless comparisons with test results show that something else is necessary the influence of concrete strength will be represented by $f_c^{1/2}$ (as in Andreasen[7], Nielsen[8], Tepfers[9] and Morita and Fujii[10] as well as Orangun, ACI 318 and BS8110) with the possibility of an upper limit for f_c.

4. The empirical equations impose various upper limits on either f_{bu} (f_{bk}) or on components of it. Whether these are intended as actual limits of bond strength or just as values beyond which the equations either become unsatisfactory or have not been verified is uncertain.

The upper limit values of f_{bk} or f_{bu} for bottom bars are:

BS8110 $f_{bk} = 0.78\sqrt{f_c} \leq 4.41 \quad N/mm^2$

EC2 ($\varphi \leq 32mm$) $f_{bk} = 0.675 f_c^{2/3}$ for $f_{ck} \leq 60 N/mm^2$

ACI 318 $f_{bk} = 1.0625\sqrt{f_{ck}}$ for $\varphi \leq 19mm$

$f_{bk} = 0.85\sqrt{f_{ck}}$ for $\varphi \leq 22mm$

Orangun et al. $f_{bu} \leq 2.12\sqrt{f_c}$

Darwin et al. $f_{bu} \leq approx. 2.8 f_c^{1/4}$

Fig.(8) compares these expressions with bond strengths obtained in tests with high resistances to splitting provided by either large concrete covers or transverse pressure (associated in one series with limited transverse reinforcement). The results considered are generally from works reviewed above but some results from Shin and Choi[43] have been added to provide data for the range $45 \leq f_c \leq 90 N/mm^2$. The principal characteristics of the specimens are tabulated in Table (3).

Table (3) Data for test results plotted in Fig.(8)

Source	φ (mm)	Test type	$\frac{c_d}{\varphi}$	$\frac{l_b}{\varphi}$	$\frac{p_u}{f_{bu}}$
Jensen[11], series 133	16	beam end	1.5	8.0	0.80-1.05
Magnusson[12]	16	pull-out	8.9	2.5	-
	20	pull-out	8.3	2.5	-
Shin & Choi[13]	13	beam end	3.9 &8.5	5.0	-
	22	beam end	5.0	5.0	-
Untrauer & Henry[14]	19	pull-out	3.5	8.0	0.40-0.80
	28	pull-out	2.2	5.3	0.22-0.93

It is clear that the upper limits that accompany the various equations should be seen as upper limits for those equations except perhaps in the case of Orangun's. They cannot be seen as actual upper limits of bond capacity for short anchorage lengths.

The anchorage lengths in the tests considered were all short, probably shorter than should be adopted in design, where the actual minimum defined by EC2 is 10φ $100mm$, which is not greatly different from the 8φ in some of the tests. Within the range $5 \leq l_b / \varphi \leq 8$ there does not appear to be any significant influence of l_b / φ on

$f_{bu,\max}$. Whether there is an influence in Magnusson's tests with $l_b / \varphi = 2.5$ depends on which part of Fig.(8) is considered.

From the figure it appears that the best safe estimate of an upper limit of bond strength would be $f_{bu,\max}$ = $approx.0.4 f_c$ although this appears to underestimate the resistance possible at very low values of

$f_c \left(< 20N / mm^2\right)$.

In view of the above the present upper limits on most equations for bond resistance seem irrelevant in the circumstances of straight end anchorages at simple supports, i.e. short bond lengths and transverse pressure.

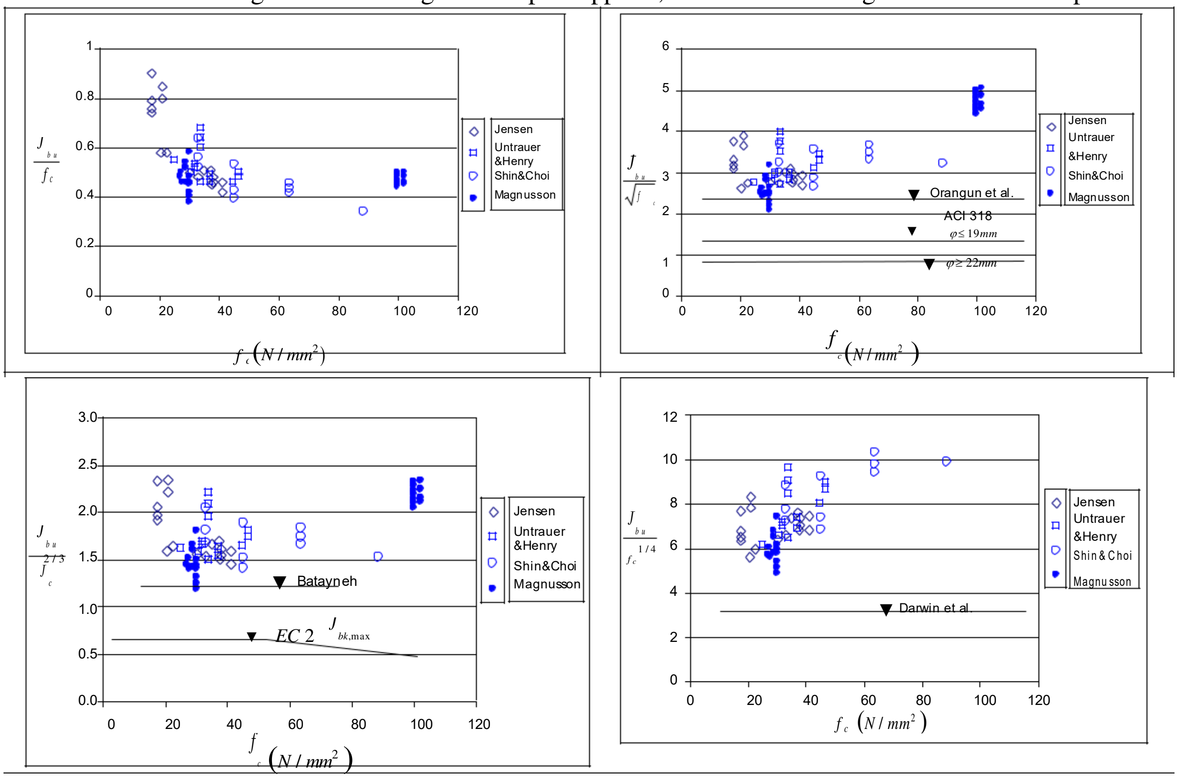


Fig.(8) Comparisons of test results with various formulations for maximum bond strength

Darwin's approach seems illogical, as the most probable reason for the average bond stress along an anchorage increasing as l_b / φ decreases is that the distribution of the bond stress becomes more uniform. It would be consistent with this for the maximum bond stress for a given concrete strength, cover and transverse reinforcement to be multiplied by a factor, which decreases as l_b / φ increases. The factor should have its maximum at the greatest value of l_b / φ for which a uniform distribution can be obtained. Nielsen employs a factor $\sqrt{\varphi / l_b}$, applied to the concrete terms in expressions for f_{bu} . This is probably a simplification as one would expect the distribution of f_b to be more uniform in the presence of stirrups and probably also more uniform for weaker (more ductile) concretes. It does however seem to be an improvement on the formulations of Orangun and Darwin. An upper limit on φ / l_b is almost certainly needed. Fig.(8) shows little or no influence of $\varphi / l_b < 0.125$.

6. CONCLUSIONS

The main conclusions drawn from the treatment of anchorage of straight bars by codes of practice and some of empirical equations in literature are as folows:

1.BS8110's expression was the simplest one which ignored most of parameters.
2. EC2's expression is the only one here to take account of transverse pressure.
3. EC2 and Darwin et al. express the influence of transverse reinforcement in terms of its area, while Orangun et al. and ACI 318 use the product of the area and yield stress. The former approach seems the more plausible where tensile resistances of steel and concrete are summed and Darwin[5] notes that in tests[15-17] the transverse reinforcement rarely yielded in failures of either splices or development lengths.
4. The ratio l_b / φ is taken into account in the equations by Orangun and Darwin though not in any of the three codes. In both equations a term proportional to φ / l_b is added to terms depending on the cover and the transverse reinforcement, and an upper limit is imposed on φ / l_b ($\varphi / l_b \leq 0.1$ in Orangun's and $\varphi / l_b \leq 0.0625$ in Darwin's). Within limits, test data appear to show that reasonable results are obtained in this way, although Fig.(8) shows that Darwin's method would give very poor results at lower ratios of l_b / φ.
5. Beyond specifications of minimum values for relative rib areas, the only empirical equation treating the influence of the geometry of deformations on bars' bond strengths is Darwin's. This allows an increase in f_{bu} with increasing f_R in the term "$13.8\, t_r t_d \sum A_{st} / s_t n\varphi$" in which $t_r = 1 + 34 f_R$ and $t_d = 1 + 0.1\varphi$. Although there is some support for this in the University of Kansas tests referred to in Ref.(18), the formulation is not well supported in other works[9,13, 19,20,21 and 22]. In general the relationship between f_{bu} and the details of bar deformations seems rather unclear. There is also the problem that in considering the results of tests from the literature, details of deformations and values of f_R are seldom given. Descriptions of deformations as complying with a specification cannot be interpreted as meaning that f_R was equal to the minimum permitted.
6. There is considerable variation in the treatment of the influence of concrete strength with bond strength being taken to be proportional to anything from $f_c^{1/4}$ to $f_c^{2/3}$, the latter applying only to $f_{ck} \leq 50N/mm^2$.
7. There are many empirical equations for bond strength and as their reliability can only be judged in relation to test results.

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