# On the Classical Primary Radical Formula and Classical Primary Subsemimodules

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**ABSTRACT—** ***In this paper, we characterize the classical primary radical of subsemimodules and classical primary subsemimodules of semimodules over a commutative semirings. Furthermore we prove that if*** $N_j$ ***is a classical primary subsemimodule of*** $M_j$***, then*** $N_j$ ***is to satisfy the classical primary radical formula in*** $M_j$ ***if and only if*** $M_1 \times M_2 \times \cdots \times M_{j-1} \times N_j \times M_{j+1} \times \cdots \times M_n$ ***is to satisfy the classical primary radical formula in*** $M$ .

**Keywords—** classical primary subsemimodule, primary subsemimodule, classical primary radical, classical primary radical formula.

## 1. INTRODUCTION

Throughout this paper a semiring will be defined as follows: A semiring is a set $R$ together with two binary operations called addition " + " and multiplication " · " such that $(R,+)$ is a commutative semigroup and $(R,\cdot)$ is semigroup; connecting the two algebraic structures are the distrubutive laws : $a(b+c)=ab+ac$ and $(a+b)c = ac+bc$ for all $a, b, c \in R$. A semimodule $M$ over a semiring $R$ is a commutative monoid $M$ with additive identity 0, together with a function $R\times M \to M$, defined by $(r, m) \mapsto rm$ such that:

1. $r(m+n) = rm + rn$
2. $(r+s)m = rm + sm$
3. $(rs)m = r(sm)$
4. $r0 = 0 = 0m$
5. $1m = m$

for all $m, n \in M$ and $r, s \in R$. Clearly every ring is a semiring and hence every module over a ring $R$ is a left semimodule over a semiring $R$. A nonempty subset $N$ of a $R$-semimodule $M$ is called subsemimodule of $M$ if $N$ is closed under addition and closed under scalar multiplication.

J. Saffar Ardabili S. Motmaen and A. Yousefian Darani in (2011) defined a different class of subsemimodules and called it classical prime. A proper subsemimodule $N$ of $M$ is said to be classical prime when for $a, b \in R$ and $m \in M$, $abm \in N$ implies that $am \in N$ or $bm \in N$.

A proper subsemimodule $N$ of $M$ is said to be classical primary when for $a, b \in R$ and $m \in M$, $abm \in N$ implies that $am \in N$ or $b^n m \in N$, for some positive integer $n$. A classical primary radical of $N$ in $M$, denoted by $c.prad_M(N)$, is defined to be the intersection of all classical primary subsemimodules containing $N$. Should there be no classical primary subsemimodule of $M$ containing $N$, then we put $c.prad_M(N) = M$. In this note, we shall need the notion of the envelope of a submodule introduced by R. L. McCasland and M. E. Moore in [11]. For a submodule $N$ of an $R$-module $M$, the envelope of $N$ in $M$, denoted by $E_M(N)$, is defined to be the subset $\{rm : r \in R$ and $m \in M$ such that $r^k m \in N$ for some $k \in \mathbb{Z}^+\}$ of $M$. Note that, in general, $E_M(N)$ is not an $R$-module. With the help of envelopes, the notion of the radical formula is defined as follows: a submodule $N$ of an $R$-module $M$ is said to satisfy the radical formula in $M$, if $\langle E_M(N) \rangle = rad_M(N)$. Also, an $R$-module $M$ is said to satisfy the radical formula, if every submodule of $M$ satisfies the radical formula in $M$. The radical formula has been studied extensively by various authors (see [8], [13] and [14]).

In this paper we introduce the concept of the radical formula and study some basic properties of this class of subsemimodules. Moreover, we prove that if $N_j$ is a classical primary subsemimodule of $M_j$, then $N_j$ is to satisfy the classical primary radical formula in $M_j$ if and only if $M_1 \times M_2 \times \cdots \times M_{j-1} \times N_j \times M_{j+1} \times \cdots \times M_n$ is to satisfy the classical primary radical formula in $M$.

## 2. PRELIMINARIES

Let $R = \prod_{i=1}^{n} R_i$, where each $R_i$ is a commutative semiring with identity. Then an ideal $I = \prod_{i=1}^{n} I_i$ of $P$ is primary if and only if $I_i$ is equal to the corresponding semiring $R_i$ and the other is primary. Moreover, any $R$-semimodule $M$ can be uniquely decomposed into a direct product of semimodules, i.e. $M = \prod_{i=1}^{n} M_i$, where

$$M_i = (0,0,0,\cdots,0,1,0,\cdots 0)M$$

is an $R_i$-semimodule with action $(r_1, r_2, \cdots, r_n)(m_1, m_2, \cdots, m_n) = (r_1 m_1, r_2 m_2, \cdots, r_n m_n)$, where $r_i \in R_i$ and $m_i \in M_i$.

**Proposition 2.1.** Let $N = N_1 \times N_2$ be a subsemimodule of $M$. Then $\langle E_M(N) \rangle = \langle E_{M_1}(N_1) \rangle \times \langle E_{M_2}(N_2) \rangle$.

**Proof.** Let $x = \sum_{i=1}^{k} (r_i, s_i)(m_i, n_i) \in \langle E_M(N) \rangle$ where $(r_i, s_i)^{k_i}(m_i, n_i) \in N$, for some $k_i \in \mathbb{Z}^+$ if and only if

$$u = \sum_{i=1}^{k} r_i m_i \in \langle E_{M_1}(N_1) \rangle, \text{ with } r_i^{k_i} m_i \in N_1$$

and

$$v = \sum_{i=1}^{k} s_i n_i \in \langle E_{M_2}(N_2) \rangle, \text{ with } s_i^{k_i} n_i \in N_2.$$

Then $x = (u, v) \in \langle E_M(N) \rangle$ if and only $u \in \langle E_{M_1}(N_1) \rangle$ and $v \in \langle E_{M_2}(N_2) \rangle$ as required.

**Corollary 2.2.** Let $N=\prod_{i=1}^{n} N_i$ be a subsemimodule of $M$. Then $\langle E_M(N)\rangle = \prod_{i=1}^{n}\langle E_{M_i}(N_i)\rangle$.

**Proof.** This follows from Proposition 2.1

**Proposition 2.3.** If $N$ is a classical prime subsemimodule of $M$, then $\langle E_M(N)\rangle = N$.

**Proof.** Clearly, $N\subseteq\langle E_M(N)\rangle$. To show that $\langle E_M(N)\rangle\subseteq N$. Let $x=\sum_{i=1}^{k} r_i m_i \in\langle E_M(N)\rangle$, where $r_i^{k_i} m_i \in N$ for some $k_i \in \mathbb{Z}^+$. Since $N$ is a classical prime subsemimodule of $M$, we have $r_i m_i \in N$. Then $x=\sum_{i=1}^{k} r_i m_i \in N$ so that $\langle E_M(N)\rangle\subseteq N$. Hence $\langle E_M(N)\rangle = N$.

## 3. CLASSICAL PRIMARY SUBSEMIMODULES

In this section, we give some characterizations for classical primary subsemimodules of $R$-semimodule $M$.

**Lemma 3.1.** Let $M=M_1\times M_2$, where $M_i$ is an $R_i$-semimodule. A subsemimodule $N_1\times M_2$ is a classical primary subsemimodule of $M$ if and only if $N_1$ is a classical primary subsemimodule of $M_1$.

**Proof.** Suppose that $N_1\times M_2$ is a classical primary subsemimodule of $R$-semimodule $M$. We will show that $N_1$ is a classical primary subsemimodule of $M_1$. Clearly, $N_1$ is a proper subsemimodule of $R_1$-semimodule $M_1$. To show that classical primary subsemimodule properties of $N_1$ hold, $m\in M_1$ and $a, b\in R_1$ such that $abm\in N_1$. Then

$$(a,1)(b,1)(m,n)=(abm,n)\in N_1\times M_2.$$

Since $N_1\times M_2$ is a classical primary subsemimodule of $R$-semimodule $M$, it follows that

$$(am,n)=(a,1)(m,n)\in N_1\times M_2$$

or

$$(b^n m,n)=(b,1)^n(m,n)\in N_1\times M_2,$$

for some positive integer $n$. That is, $am\in N_1$ or $b^n m\in N_1$. Therefore $N_1$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$. Conversely, suppose that $N_1$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$. We will show that $N_1\times M_2$ is a classical primary subsemimodule of $R$-semimodule $M$. Clearly, $N_1\times M_2$ is a proper subsemimodule of $R$-semimodule $M$. To show that classical primary subsemimodule properties of $N_1\times M_2$ hold, let $(m,n)\in M$ and $(a_1,a_2), (b_1,b_2)\in R$ such that

$$(a_1b_1m, a_2b_2n)=(a_1,a_2)(b_1,b_2)(m,n)\in N_1\times M_2.$$

Since $N_1$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$ and $a_1b_1m\in N_1$, we have $a_1m\in N_1$ or $b_1^n n\in N_1$, for some positive integer $n$. Therefore

$$(a_1,a_2)(m,n)=(a_1m,a_2n)\in N_1\times M_2$$

or

$$(b_1,b_2)^n(m,n)=(b_1^n m, b_2^n n)\in N_1\times M_2.$$

Hence $N_1\times M_2$ is a classical primary subsemimodule of $R$-semimodule $M$.

**Corollary 3.2.** Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. A subsemimodule $M_1 \times N_2$ is a classical primary subsemimodule of $R$-semimodule $M$ if and only if $N_2$ is a classical primary subsemimodule of $R_2$-semimodule $M_2$.

**Proof.** This follows from Lemma 3.1.

**Corollary 3.3.** Let $M = \prod_{i=1}^{n} M_i$, where $M_i$ is an $R_i$-semimodule. A subsemimodule

$$M_1 \times M_2 \times \cdots \times M_{j-1} \times N_j \times M_{j+1} \times \cdots \times M_n$$

is a classical primary subsemimodule of $R$-semimodule $M$ if and only if $N_j$ is a classical primary subsemimodule of $R_j$-semimodule $M_j$.

**Proof.** This follows from Lemma 3.1 and Corollary 3.2.

**Lemma 3.4.** Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. If $N_1 \times \{n\}$ is a classical primary subsemimodule of $M$, then $N_1$ is a classical primary subsemimodule of $M_1$.

**Proof.** Suppose that $N_1 \times \{n\}$ is a classical primary subsemimodule of $R$-semimodule $M$. We will show that $N_1$ is a classical primary subsemimodule of $M_1$. Clearly, $N_1$ is a proper subsemimodule of $R_1$-semimodule $M_1$. To show that classical primary subsemimodule properties of $N_1$ hold, let $m \in M_1$ and $a, b \in R_1$ such that $abm \in N_1$. Then

$$(a,1)(b,1)(m,n) = (abm, n) \in N_1 \times \{n\}.$$

Since $N_1 \times M_2$ is a classical primary subsemimodule of $R$-semimodule $M$, it follows that

$$(am, n) = (a,1)(m,n) \in N_1 \times \{n\}.$$

or

$$(b^n m, n) = (b,1)^n (m,n) \in N_1 \times \{n\}.$$

for some positive integer $n$. That is, $am \in N_1$ or $b^n m \in N_1$. Therefore $N_1$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$.

**Corollary 3.5.** Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. If $\{n\} \times N_2$ is a classical primary subsemimodule of $R$-semimodule $M$, then $N_2$ is a classical primary subsemimodule of $R_2$-semimodule $M_2$.

**Proof.** This follows from Lemma 3.4.

**Corollary 3.6.** Let $M = \prod_{i=1}^{n} M_i$, where $M_i$ is an $R_i$-senimodule. If $\{m_1\} \times \{m_2\} \times \cdots \times N_j \times \cdots \times \{m_n\}$ is a classical primary subsemimodule of $R$-semimodule $M$, then $N_j$ is a classical primary subsemimodule of $R_j$-semimodule $M_j$.

**Proof.** This follows from Lemma 3.4 and Corollary 3.5.

## 4. RADICAL OF CLASSICAL PRIMARY SUBSEMIMODULES

A subsemimodule $N$ of an $R$-semimodule $M$ is said to satisfy the classical primary radical formula in $M$, if $\langle E_M(N)\rangle = \text{c}.\, prad_M(N)$.

**Lemma 4.1.** Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. If $W$ is a classical primary subsemimodule of $R$-semimodule $M$ and $P = \{x \in M_1 : (x, y) \in W\}$, then $P = M_1$ or $P$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$.

**Proof.** Suppose that $P \neq M_1$. We will show that $P$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$. It is clear that, $P$ is a proper subsemimodule of $R_1$-semimodule $M_1$. To show that classical primary subsemimodule properties of $P$, let $a, b \in R_1$ and $m \in M_1$ such that $abm \in P$. Then $(a,1)(b,1)(m, y) = (abm, y) \in W$. Since $W$ is a classical primary subsemimodule of $M$, we have

$$(am,1) = (a,1)(m, y) \in W$$

or

$$(b^n m, y) = (b,1)^n (m, y) \in W,$$

for some positive integer $n$. It follows that $am \in P$ or $b^n m \in P$. Therefore $P$ is a classical primary subsemimodule of $M_1$.

**Corollary 4.2**. Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. If $W$ is a classical primary subsemimodule of $R$-semimodule $M$ and $P = \{x \in M_2 : (0, x) \in W\}$, then $P = M_2$ or $P$ is a classical primary subsemimodule of $R_2$-semimodule $M_2$.

**Proof.** This follows from Lemma 4.1.

**Corollary 4.3.** Let $M = \prod_{i=1}^{n} M_i$, where $M_i$ is an $R_i$-semimodule. If $W$ is a classical primary subsemimodule of $R$-semimodule $M$ and $P = \{x \in M_j : (m_1, m_2, \cdot, x, m_{j+1}, \cdot, m_n) \in W\}$, then $P = M_j$ or $P$ is a classical primary subsemimodule of $R_j$-semimodule $M_j$.

**Proof.** This follows from Lemma 4.1 and Corollary 4.2.

**Lemma 4.4.** Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule and let $N$ be a subsemimodule of $R_1$-semimodule $M_1$. Then $m \in c.\, prad_{M_1}(N)$ if and only if $(m, y) \in c.\, prad_{M_1}(N \times \{y\})$.

**Proof.** Suppose that $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. Let $N$ be a subsemimodule of $R_1$-semimodule $M_1$ and let $m \in c.\, prad_{M_1}(N)$.

If there is no classical primary subsemimodule of $M$ containing $N \times \{y\}$, then $c.\, prad_M(N \times \{y\}) = M$. Therefore $(m, y) \in c.\, prad_M(N \times \{y\})$.

If there is classical primary subsemimodule of $M$ containing $N \times \{y\}$, then there exists a classical primary subsemimodule $W$ with $N \times \{y\} \subseteq W$. By Lemma 4.1 and $P = \{x \in M_1 : (x, y) \in W\}$, we have $P = M_1$ or $P$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$.

**Case 1:** $P = M_1$. Since $m \in c.prad_{M_1}(N)$, we have $m \in P$. Then $(m, y) \in W$. Therefore if $W$ is a classical primary subsemimodule of $M$ containing $N \times \{y\}$, then $(m, y) \in W$.

**Case 2:** $P \neq M_1$. Since $P \neq M_1$, we have $P$ is a classical primary subsemimodule of $R_1$-semimodule $M_1$. Let $x \in N$. Then $(x, y) \in N \times \{y\}$ so that $x \in P$. It follows that $N \subseteq P$. We have

$$\begin{aligned} c.rad_{M_1}(N) &\subseteq c.rad_{M_1}(P) \\ &= P \end{aligned}$$

so that $m \in P$. Therefore if $W$ is a classical primary subsemimodule of $M$ containing $N \times \{y\}$, then $(m, y) \in W$ and hence $(m, y) \in c.prad_{M_1}(N \times \{y\})$.

**Corollary 4.5**. Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule and let $N$ be a subsemimodule of $R_2$-semimodule $M_2$. Then $m \in c.prad_{M_2}(N)$ if and only if $(x, m) \in c.prad_M(\{x\} \times N)$.

**Proof.** This follows from Lemma 4.4.

**Corollary 4.6** Let $M = \prod_{i=1}^{n} M_i$, where $M_i$ is an $R_i$-semimodule and let $N$ be a subsemimodule of $R_j$-semimodule $M_j$. Then $m \in c.prad_{M_j}(N)$ if and only if

$$(x_1, \cdots, m, x_{j+1}, \cdots, x_n) \in c.prad_M(\{x_1\} \times \{x_2\} \times \cdots \times N \times \{x_{j+1}\} \times \cdots \times \{x_n\}).$$

**Proof.** This follows from Lemma 4.4 and Corollary 4.5.

**Lemma 4.7.** Let $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. If $N_i$ be a subsemimodule of $R_i$-semimodule $M_i$, then $c.prad_{M_1}(N_1) \times c.prad_{M_2}(N_2) \subseteq c.prad_M(N_1 \times N_2)$.

**Proof.** Suppose that $M = M_1 \times M_2$, where $M_i$ is an $R_i$-semimodule. Let $N_1$ be a subsemimodule of $R_1$-semimodule $M_1$. We will show that $c.prad_{M_1}(N_1) \times .cprad_{M_2}(N_2) \subseteq c.prad_M(N_1 \times N_2)$. Let

$$(x, y) \in c.prad_{M_1}(N_1) \times c.prad_{M_2}(M_2).$$

Then $x \in c.prad_{M_1}(N_1)$ and $y \in c.prad_{M_2}(N_1)$. By Lemma 4.1 and Lemma 4.4, we have

$$(x, 0) \in c.prad_M(N_1 \times \{0\}) \subseteq c.prad_M(N_1 \times N_2)$$

and

$$(0, y) \in c.prad_M(\{0\} \times N_2) \subseteq c.prad_M(N_1 \times N_2).$$

Then $(x, y) = (x, 0) + (0, y) \in c.prad_M(N_1 \times N_2)$ and hence

$$c.prad_{M_1}(N_1) \times c.prad_{M_2}(N_2) \subseteq c.prad_M(N_1 \times N_2).$$

**Corollary 4.8**. Let $M=\prod_{i=1}^{n}M_i$, where $M_i$ is an $R_i$-semimodule. If $N_i$ be a subsemimodule of $R_i$-semimodule $M_i$, then $\prod_{i=1}^{n} c.prad_{M_i}(N_i)\subseteq c.prad_M(\prod_{i=1}^{n}N_i)$.

**Proof.** This follows from Lemma 4.7.

**Theorem 4.9.** Let $M=M_1\times M_2$, where $M_i$ is an $R_i$-semimodule. If $N$ is a subsemimodule of $R_1$-semimodule $M_1$, then $c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2)=c.prad_M(N_1\times M_2)$.

**Proof.** Suppose that $M=M_1\times M_2$, where $M_i$ is an $R_i$-semimodule. Let $N$ be a subsemimodule of $R_1$-semimodule $M_1$. By Lemma 4.7, we have $c.prad_{M_1}(N)\times c.prad_{M_2}(M_2)\subseteq c.prad_M(N\times M_2)$. We will show that $c.prad_M(N_1\times M_2)\subseteq c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2)$. If there is no classical primary subsemimodule of $M$ containing $N$, then $c.prad_{M_1}(N)=M_1$. Then

$$c.prad_M(N_1\times M_2)\subseteq c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2).$$

If there is classical primary subsemimodule of $M$ containing $N$, then there exists $W$ is a classical primary subsemimodule of $M_1$ containing $N$. Then $W\times M_2$ is a classical primary subsemimodule of $M$, containing $N\times M_2$. Let $P$ be a classical primary subsemimodule of $M$ containing $N\times M_2$. Then

$$\begin{aligned} N\times M_2 &\subseteq c.prad_{M_1}(N)\times M_2 \\ &= c.prad_{M_1}(N)\times c.prad_{M_2}(M_2). \end{aligned}$$

Therefore $c.prad_M(N_1\times M_2)\subseteq c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2)$ and hence

$$c.prad_M(N_1\times M_2)=c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2).$$

**Corollary 4.10**. Let $M=M_1\times M_2$, where $M_i$ is an $R_i$-semimodule. If $N$ is a subsemimodule of $R_2$-semimodule $M_2$, then $c.prad_M(M_2\times N)=c.prad_{M_1}(M_2)\times c.prad_{M_2}(N)$.

**Proof.** This follows from Lemma 4.9.

**Corollary 4.11.** Let $M=\prod_{i=1}^{n}M_i$, where $M_i$ is an $R_i$-semimodule. If $N_i$ be a subsemimodule of $R_i$-semimodule $M_i$, then $\prod_{i=1}^{n} c.prad_{M_i}(N_i)=c.prad_M(\prod_{i=1}^{n}N_i)$.

**Proof.** This follows from Lemma 4.9 and Corollary 4.10.

**Theorem 4.12**. Let $M=M_1\times M_2$, where $M_i$ is an $R_i$-semimodule. If $N_1$ is a classical primary subsemimodule of $M_1$, then $N_1$ is to satisfy the classical primary radical formula in $M_1$ if and only if $N_1\times M_2$ is to satisfy the classical primary radical formula in $M$.

**Proof.** Suppose that $N_1$ is a classical primary subsemimodule of $M_1$ and $N_1$ is to satisfy the classical primary radical formula in $M_1$. We will show that $N_1\times M_2$ is to satisfy the classical primary radical formula in $M$. Since $N_1$ is a classical primary subsemimodule of $M_1$, it follows that

$$c.\,prad_M(N_1\times M_2) = c.\,prad_{M_1}(N_1)\times c.\,prad_{M_2}(M_2)$$
$$= \langle E_{M_1}(N_1)\rangle \times M_2$$
$$= E_M(N_1\times M_2).$$

Therefore $N_1 \times M_2$ is to satisfy the classical primary radical formula in $M$. Conversely, suppose that $N_1$ is a classical primary subsemimodule of $M_1$ and $N_1\times M_2$ is to satisfy the classical primary radical formula in $M$. We will show that $N_1$ is to satisfy the classical primary radical formula in $M_1$. Since $N_1\times M_2$ is a classical primary subsemimodule of $M$, it follows that

$$\langle E_{M_1}(N_1)\rangle \times M_2 = \langle E_M(N_1\times M_2)\rangle$$
$$= c.\,prad_{M_1}(N_1)\times c.\,prad_{M_2}(M_2).$$

Then $c.\,prad_{M_1}(N_1) = \langle E_{M_1}(N_1)\rangle$ and hence $N_1$ is to satisfy the classical primary radical formula in $M_1$.

**Corollary 4.13**. Let $M = \prod_{i=1}^{n} M_i$, where $M_i$ is an $R_i$-semimodule. If $N_j$ is a classical primary subsemimodule of $M_j$, then $N_j$ is to satisfy the classical primary radical formula in $M_j$ if and only if

$$M_1\times M_2\times\ldots\times M_{j-1}\times N_j\times M_{j+1}\times\ldots\times M_n$$

is to satisfy the classical primary radical formula in $M$.

**Proof.** This follows from Theorem 4.12.


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